Nonlinear wave interactions as a source of energy on the seafloor

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Introduction

- Nonlinear interactions among multidirectional waves in deep water → acoustic gravity waves into the water column → can excite resonances in the coupled compressible ocean- elastic seafloor system; ⇒ microseisms. (Longuet-Higgins, Hasselmann, Webb, Kibblewhite, Elgar, Herbers, Guza, and others.)
- Similar effect occurs in shallow water; nonlinear interactions between swells and a sloping seafloor (Hasselmann, Ardhuin).
- is there enough energy in the acoustic-gravity waves for seafloor instruments?

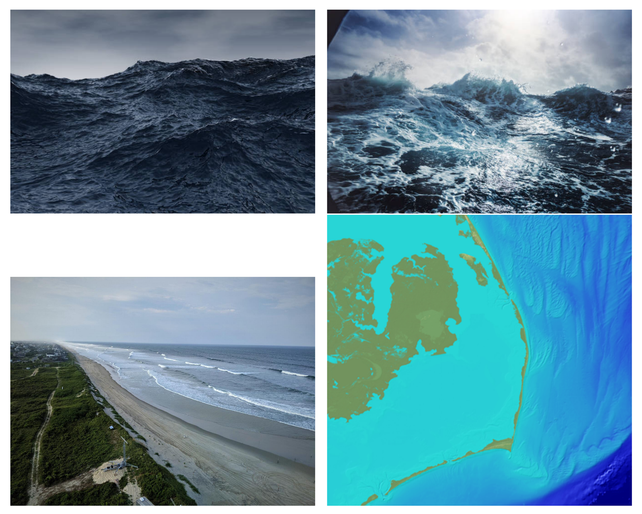

Figure 1. Top row: multi-directional 'confused' seas: mid-ocean; Bottom row: waves over shoaling seafloor (e.g., Outer Banks, NC.)

Deep water: Excitation by interacting surface waves

- Treatment follows Hasselmann [1]; more general than works published since.
- Second-order pressure spectrum S_E due to surface-wave spectrum S_ζ,

$$S_E(\boldsymbol{k},\omega) = \rho^2 g^4 \iint_{\boldsymbol{\kappa},\sigma} \iint_{\boldsymbol{\kappa}',\sigma'} S_\zeta(\boldsymbol{\kappa},\sigma) S'_\zeta(\boldsymbol{\kappa}',\sigma')(\kappa\kappa' - \boldsymbol{\kappa}\boldsymbol{\kappa}')^2 \frac{1}{(\sigma\sigma')^2} \cdots$$
$$\times [\delta(\boldsymbol{k} - (\boldsymbol{\kappa}+\boldsymbol{\kappa}'))][\delta(\omega - (\sigma+\sigma'))] d\boldsymbol{\kappa} d\boldsymbol{\kappa}' d\sigma d\sigma'. \qquad (1)$$

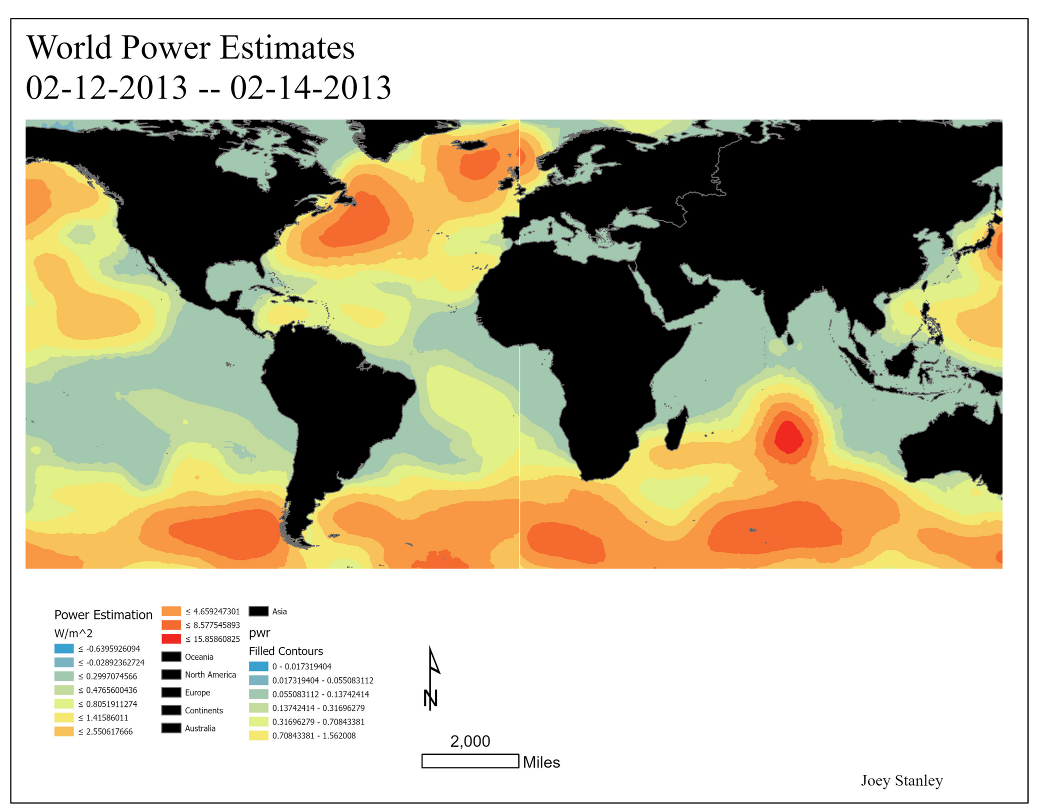

Figure 2. Estimated power density (W/m²) based on theory. Courtesy: Joey Stanley, JHU.

$$S_A(\boldsymbol{k},t) = \sum_{n=1}^{N_G} \frac{\pi(t_{ne}-t_{ns})D_1^2}{2\omega_1^2} S_E(\omega_1,\boldsymbol{k}) + \int_{t_{ne}}^{t} \frac{S_A(\boldsymbol{k},\tau)}{\tau} d\tau$$
$$S_P(\boldsymbol{k},\omega) = \frac{dP_1 dP_1^*}{d\boldsymbol{k} d\omega} = \left(\lambda\gamma^2\right)^2 S_A(\boldsymbol{k},\omega), \qquad (2)$$

Power density (W/m^2) at the seafloor:

$$\Pi(\boldsymbol{k},\omega) = \sqrt{S_W(\boldsymbol{k},\omega) d\boldsymbol{k} d\omega};\; S_W(\boldsymbol{k},\omega) = \frac{S_P(\boldsymbol{k},\omega)}{\rho c_1}. \qquad (3)$$

Resonant energy transfer: surface waves to seafloor

- Finite depth compressible ocean over semi-infinite elastic seafloor, coupled through interface/boundary conditions.
- Underwater wave groups comprised of acoustic-gravity waves in water and Scholte waves on seafloor [2].
- When $(\sigma, \boldsymbol{\kappa})$ in (1) → $(\omega, \boldsymbol{k})$ ∈ a dispersion surface defined by the interface conditions [2] → resonant energy transfer from surface to seafloor.
- ⇒ for small dissipation, forced second-order spectrum grows linearly with time, though 'interaction times' may be short.
- Forced wave number spectra for pressure; multiple generation areas:

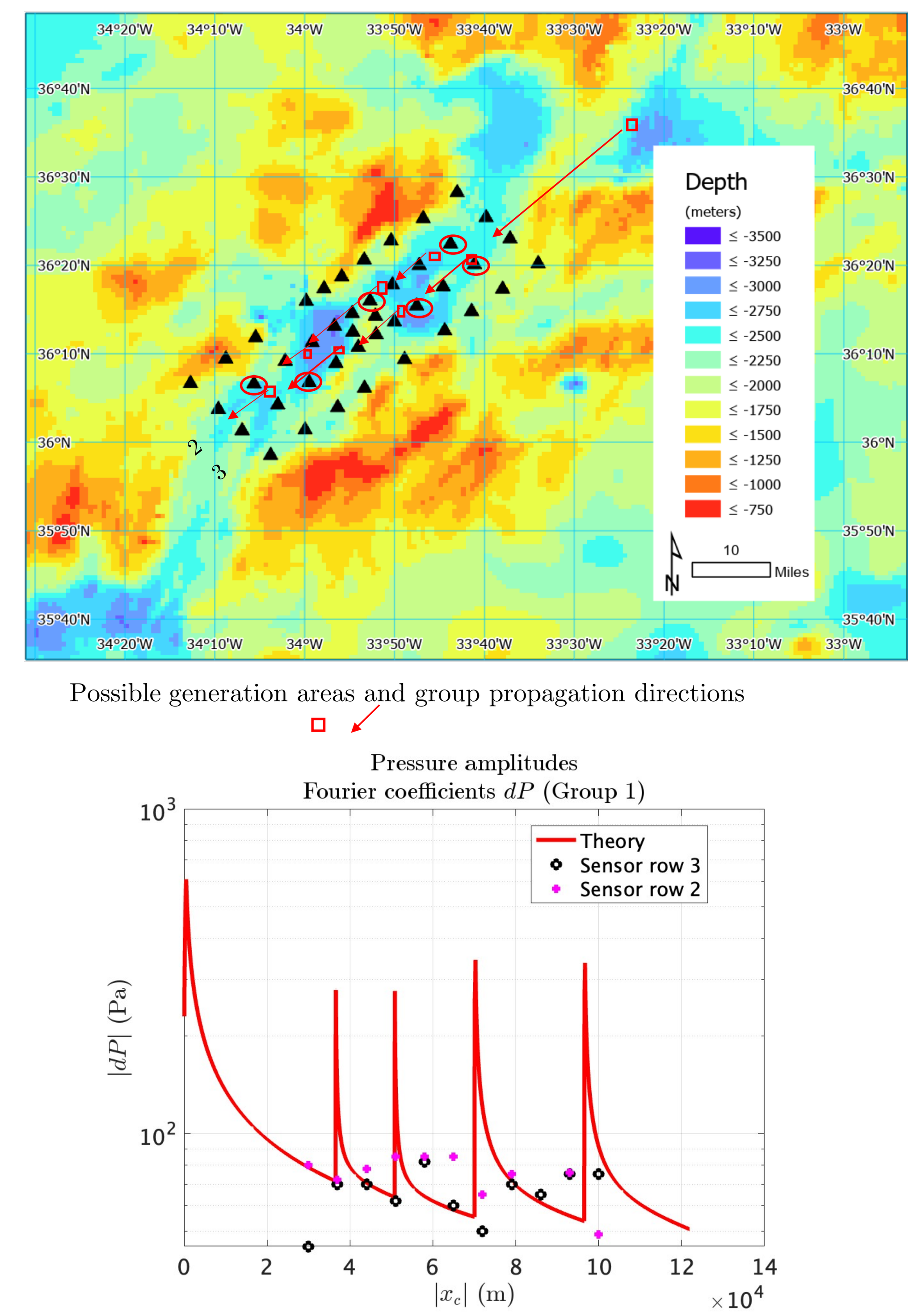

Figure 3. Scripps array mid-Atlantic at ∼ 34°N, 36°W [3], and predicted (curve) + measured (symbols) pressures.

Shallow water wave-seafloor interactions

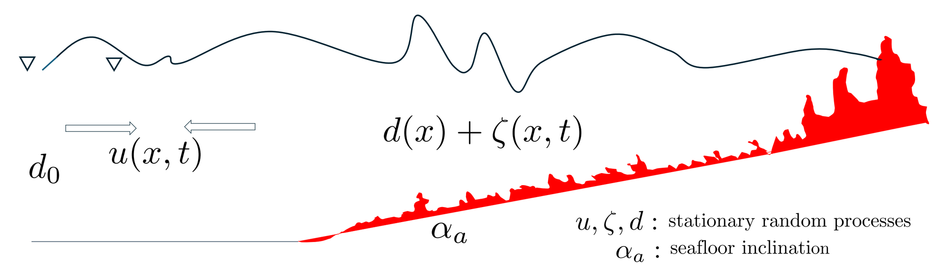

Figure 4. Schematic view of the interactions being studied.

- Waves interact with the seafloor and with their own reflections from shore. Solution for random wave inputs developed using Fourier-Stieltjes integrals.
- Boussinesq equations (with reflections) with nonlinearity and dispersion used [4].
- Equations first normalized, then expanded in a perturbation series in $\alpha = A/d_0$, A being the predominant wave amplitude and d_0 the water depth at shelf bottom.

Shallow water results

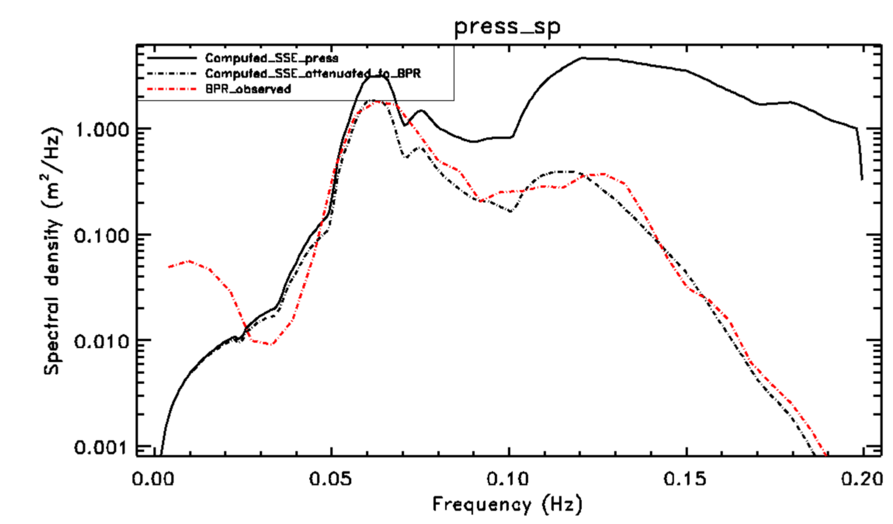

Figure 5. Spectral density vs. frequency for computed sea surface elevation pressure field (black curve) computed sea-surface elevation spectrum converted to equivalent bottom pressure at 31-m depth (dashed-black), and observed pressure at the bottom in 31-m depth (dashed-red). Date: 12/15/21, 00:30 hrs. 32.9°N, 117.3°W, depth = 31m.

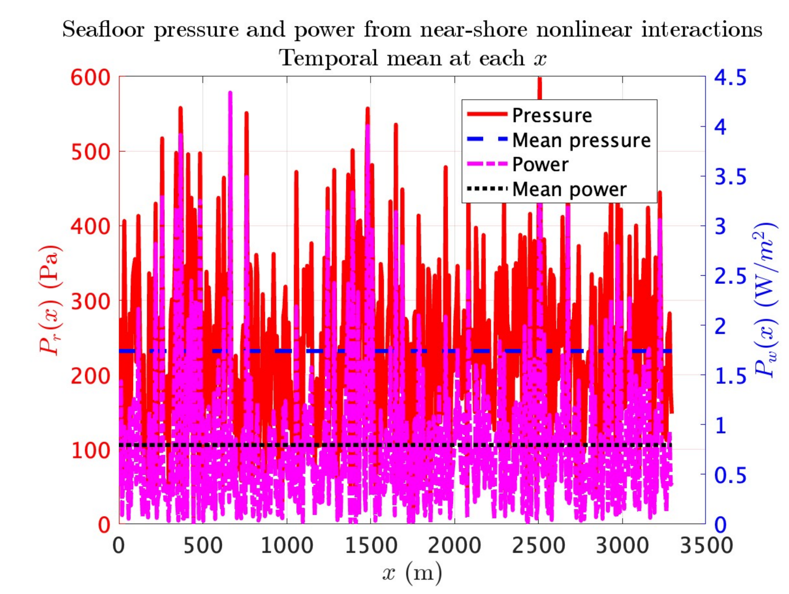

Figure 6. Pressure and power density at the undisturbed free surface due to the shallow-water random waves at 31m depth (surface $H_s = 2.6$m).

Conclusions

- Seafloor power densities in the range 5–35 W/m^2 estimated in some parts of the ocean in deep water.
- Shallow water power densities approach 1 W/m^2 for surface significant wave heights H_s = 2.6m.
- Extensions to enable shallow-water microseism amplitudes in progress.

References

[1] K. Hasselmann.
A statistical analysis of the generation of microseisms.
Reviews of Geophysics, 1(2):177–210, 1963.

[2] U.A. Korde.
Propagation of underwater wave groups in a compressible ocean coupled with an elastic seafloor.
Journal of Fluid Mechanics, 2024.
in press.

[3] R.A. Dunn, R. Arai, D.E. Eason, J.C. Canales, and R.A. Sohn.
Three-dimensional seismic structure of the Mid-Atlantic Ridge: An investigation of tectonic, magmatic, and hydrothermal processes in the Rainbow area.
Journal of Geophysical Research: Solid Earth, 122:9580–9602, 2017.

[4] G.B. Whitham.
Linear and Nonlinear Waves, chapter 14.
John Wiley & Sons, NY, 1973.
A Wiley-Interscience Publication.

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